values of α_1, α_2, β_1, β_2 found from Eq. (13) and the corresponding maximum errors of the approximation δ.

The case when $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$ corresponds to approximation of the binary intermediate components on the basis of the Lichtenecker formula (7).

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DYNAMIC CONTACT PROBLEM FOR A TWO-LAYER HALF-SPACE WITH A SPHERICAL CAVITY

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This article examines the problem of exciting steady harmonic vibrations in an elastic two-layer half-space with a spherical cavity. The vibrations are excited by impact of a rigid die against the plane surface. The deformation of the medium is presumed to be axisymmetric. The boundary-value problem is solved in two steps. In the first step, we study the problem of the excitation of steady harmonic vibrations in an elastic two-layer half-space with a deep-set spherical cavity. Here, the vibrations are excited by a weightless rigid circular die with a flat base. With allowance for the radiation conditions, the boundary-value problem is reduced to a system of integrodifferential equations. These equations are studied by asymptotic methods and the method of approximate factorization of matrix functions. With the assumption that the amplitude of the load acting on the die is constant, we find the distribution law for the contact stresses and the amplitude—frequency characteristics of points of the base of the die. In the second step, we consider the effect of the mass of the die by examining the equation of its motion as a rigid mass under the influence of an assigned external load and the reaction of its complex elastic base. The reaction of the base is found from the first step for the given frequency range.

Results are presented from study of the basic laws governing the behavior of the amplitude—frequency characteristics in relation to the mass of the die and the location of the cavity in the medium for different ratios of the stiffnesses of the layer and the half-space.

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1. Let an elastic medium located in the cylindrical coordinate system (R, z, θ) occupy the region $D = D_1 \cup D_2$, where $D_1 = \{R > 0,\ z < 0,\ \theta \in (0, 2\pi),\ r = \sqrt{R^2 + (z + h)^2} > a\}$, $D_2 = \{R > 0,\ \theta \in (0, 2\pi),\ z \in (0, h_1)\}$, $\{a, h\}$ are the radius of the cavity and the depth of its center in the half-space. The properties of the medium in the region D_j are described by the parameters μ_j (shear modulus) and V_{pj}, V_{sj} (velocities of longitudinal and transverse waves).

A rigid circular weightless die with a flat base of radius b $(R < b)$ undergoes harmonic vibrations with the frequency ω on the surface of the layer $z = h_1$ under the influence of a concentrated force

$$P(R,\ t) = p\delta(R - R_0)\exp(-i\omega t). \tag{1.1}$$

The contact between the die and the layer and between the layer and the half-space is assumed to be rigid. At the boundary of the region D, outside the die, we assign conditions that correspond to the absence of stresses:

$$z = h_1\colon\ \sigma_{zz} = 0,\ \tau_{Rz} = 0,\ r = a\colon\ \sigma_{rr} = 0,\ \tau_{r\varphi} = 0. \tag{1.2}$$

Energy radiation conditions are assigned at infinity. The spherical coordinate system (r, φ, θ) $(r \in (a, \infty),\ \varphi \in (0, \pi),\ \theta \in (0, 2\pi))$ is connected with the center of the cavity. The angle φ is reckoned from the vertical in the clockwise direction. All of the relations will henceforth be written in dimensionless parameters. Linear dimensional parameters are obtained from the dimensionless parameters by multiplying them by the dimensional radius of the cavity a, while quantities having the dimension of stress are obtained by multiplying the corresponding dimensionless quantities by the shear modulus μ_1.

The amplitude functions $\mathbf{u}^{(j)}$ of the displacements of points of the region D_j are sought in the form

$$\begin{gathered}
\mathbf{u}^{(2)} = \{u_z^{(2)}, u_R^{(2)}\},\ \mathbf{u}^{(1)} = \{u_z^{(1)}, u_R^{(1)}\} = \mathbf{u}^{(1,1)} + \mathbf{u}^{(1,2)},\\
u_R = \int_\Gamma \widetilde{u}_R \alpha J_1(\alpha R)\, d\alpha,\ u_R = u_R^{(2)} \quad \text{or} \quad u_R = u_R^{(1,2)},\\
u_z = \int_\Gamma \widetilde{u}_z \alpha J_0(\alpha R)\, d\alpha,\quad u_z = u_z^{(2)} \quad \text{or}\ u_z = u_z^{(1,2)},\\
\widetilde{u}_R^{(1,2)} = \widetilde{X}_2\alpha^2 M(\alpha, z) + \widetilde{X}_1\alpha S(\alpha, z),\\
\widetilde{u}_z^{(1,2)} = -\widetilde{X}_2\alpha L(\alpha, z) + \widetilde{X}_1 R(\alpha, z),\\
\widetilde{u}_R^{(2)} = [\widetilde{R}_2\alpha^2 M_H(\alpha, z) + \widetilde{R}_1\alpha S_H(\alpha, z) + \widetilde{q}_2\alpha^2 M_b(\alpha, z) + \widetilde{q}_1\alpha S_b(\alpha, z)]\,\gamma,\\
\gamma = \mu_1/\mu_2,\\
\widetilde{u}_z^{(2)} = [-\widetilde{R}_2\alpha L_H(\alpha, z) + \widetilde{R}_1 R_H(\alpha, z) - \widetilde{q}_2\alpha L_b(\alpha, z) + \widetilde{q}_1 R_b(\alpha, z)]\,\gamma.
\end{gathered} \tag{1.3}$$

The contour Γ is drawn in accordance with the principle of limiting absorption. In the given case, it coincides with the real positive semi-axis everywhere except for the poles and the branch points of the integrand. The contour passes below these points.

Expressions for the functions M, S, L, R, M_H, S_H, M_b, S_b, L_H, R_H, L_b, R_b were presented in [1]. The tilde denotes application of the Hankel transform to the functions X_j, R_j, q_j, which correspond to the normal or shear stresses on one of the planes $z = 0$ or $z = h_1$ (the quantity α is the Hankel transform parameter):

$$\mathbf{u}^{(1,1)} = \{u_r^{(1,1)}\sin\varphi + u_\varphi^{(1,1)}\cos\varphi,\ u_r^{(1,1)}\cos\varphi - u_\varphi^{(1,1)}\sin\varphi\}. \tag{1.4}$$

The functions $u_r^{(1,1)}$, $u_\varphi^{(1,1)}$ determine the displacements in a space with a spherical cavity of unit radius in the case when the cavity surface is subjected to the assigned distributed forces $\sigma_{rr}|_{r=1} = Y_1(\varphi)$, $\tau_{r\varphi}|_{r=1} = Y_2(\varphi)$. These functions are found as follows:

$$u_r^{(1,1)} = \sum_{n=0}^{\infty}\left\{A_n \frac{d}{dr} h_n^{(1)}(\theta_{11}r) + \frac{B_n}{r} h_n^{(1)}(\theta_{12}r)\right\} P_n(\cos\varphi),$$

$$u_\varphi^{(1,1)} = \sum_{n=0}^{\infty}\left\{\frac{A_n}{r}h_n^{(1)}(\theta_{11}r) + B_n\frac{d}{dr}\left(rh_n^{(1)}(\theta_{12}r)\right)/(n(n+1)r)\right\}\frac{\partial}{\partial\varphi}(P_n(\cos\varphi)). \tag{1.5}$$

Here, $A_n = \sqrt{(2/\pi)}(Y_{1n}a_{4n} - Y_{2n}a_{2n})(\Delta_n)^{-1}$; $\Delta_n = a_{1n}a_{4n} - a_{2n}a_{3n}$; $B_n = \sqrt{(2/\pi)}(Y_{2n}a_{1n} - Y_{1n}a_{3n})\cdot(\Delta_n)^{-1}$; $h_n^{(1)}(x)$ are spherical Hankel functions; $P_n(\cos\varphi)$ are Legendre polynomials; a_{jn} are known functions of the corrected frequencies [2]. After we satisfy boundary conditions (1.2) at r = 1 (the first equation of the system) and the compatibility conditions (for the subsequent equations), we obtain a system of integrofunctional equations in X_j, R_j, q_j (j = 1, 2):

$$\begin{gathered}
\mathbf{Y}(\varphi) - \int_\Gamma \mathbf{G}(\alpha,\varphi)\widetilde{\mathbf{X}}(\alpha)\,d\alpha = 0,\ \widetilde{\mathbf{X}}(\alpha) + \widetilde{\mathbf{T}}(\alpha) = \widetilde{\mathbf{R}}(\alpha),\\
\mathbf{P}(\alpha)\widetilde{\mathbf{X}}(\alpha) + \widetilde{\mathbf{V}}(\alpha) = \gamma\mathbf{Q}(\alpha)\widetilde{\mathbf{R}}(\alpha) + \gamma\mathbf{S}(\alpha)\widetilde{\mathbf{q}}(\alpha),\\
\mathbf{V}_* = \sum_{n=0}^{\infty} i^{-n+1}\left\{\mathbf{f}A_n\exp(-h\lambda_{11})P_n\left(\frac{i\lambda_{11}}{\theta_{11}}\right) + \mathbf{g}B_n\exp(-h\lambda_{12})P_n^1\left(\frac{i\lambda_{21}}{\theta_{21}}\right)\right\},\\
\mathbf{V}_* = \{\widetilde{V}_1(\alpha), \widetilde{V}_2(\alpha), \widetilde{T}_1(\alpha), \widetilde{T}_2(\alpha)\},\\
\mathbf{X} = \{X_1, X_2\},\ \mathbf{R} = \{R_1, R_2\},\ \mathbf{Y} = \{Y_1, Y_2\},\\
\mathbf{q} = \{q_1, q_2\},\ \mathbf{V} = \{V_1, V_2\},\ \mathbf{T} = \{T_1, T_2\},\\
\mathbf{f} = \{f_k\},\ \mathbf{g} = \{g_k\}\ (k = 1, 2, 3, 4)
\end{gathered} \tag{1.6}$$

(**G**, **P**, **Q**, **S** are second-order matrix functions with the components G_{nm}, P_{nm}, Q_{nm}, S_{nm} (n, m = 1, 2), the latter themselves being functions of the Hankel transform parameter α and the elastic constants of the medium. Here, G_{nm} and P_{nm} have one pole on the positive real half-plane, while Q_{nm} and S_{nm} (n, m = 1, 2) are meromorphic functions having a finite number of poles on R^+:

$$\begin{gathered}
f_1 = \theta_{11}^{-1},\quad f_2 = -\frac{\alpha}{\theta_{11}\lambda_{11}},\quad f_3 = \frac{2\alpha^2 - \theta_{11}^4\theta_{21}^{-2}}{\theta_{11}\lambda_{11}},\quad f_4 = \frac{2\alpha}{\theta_{11}},\\
g_1 = \alpha\theta_{21}^{-1}\lambda_{21}^{-1},\ g_2 = -\theta_{21}^{-1},\ g_3 = 2\alpha\theta_{21}^{-1},\ g_4 = \frac{\alpha^2 + \lambda_{21}^2}{\theta_{21}\lambda_{21}},\\
\lambda_{ij} = \sqrt{\alpha^2 - \theta_{ij}^2},\quad \theta_{1j} = \frac{\omega a}{V_{pj}},\quad \theta_{2j} = \frac{\omega a}{V_{sj}}\quad (i,\ j = 1, 2),\\
G_{nm}(\alpha,\varphi) = \Delta_R^{-1}(\alpha)\sum_{j=1}^{2} G_{nm}^{(j)}(\alpha,\varphi)\exp(-\lambda_{j1}(-\cos\varphi + h)).
\end{gathered}$$

The expressions for $G_{nm}^{(j)}$, P_{nm}, Q_{nm}, S_{nm} are omitted here due to their awkwardness.

With allowance for the fact that the stresses q_j in the contact region are unknown, we augment system (1.6) with an equation which characterizes the equality of the displacements in the layer under the die and the assigned function $W_*(t)$; this equation closes system (1.6) and can be written as

$$\int_\Gamma [-\widetilde{R}_2\alpha L_H(\alpha) + \widetilde{R}_1 R_H(\alpha) + \widetilde{q}_1 R_b(\alpha)]\,J_0(\alpha R)\,\alpha\,d\alpha\exp(-i\omega t) = W_*(t). \tag{1.7}$$

Let us study the resulting system (1.6), (1.7). With the condition that the cavity is set deep in the half-space (h > 1), we can use Artsel's theorem [3] to show that the operator

$$\mathbf{K}\cdot\widetilde{\mathbf{X}} = \int_\Gamma \mathbf{G}(\alpha,\varphi)\cdot\widetilde{\mathbf{X}}(\alpha)\,d\alpha$$

is continuous in the space $\mathscr{L}^2((-\pi, \pi))$. With the condition h ≫ 1, it becomes small and is estimated as

$$\max_{1 \le i \le 2} \sum_{m=1}^{2} \sqrt{\int_{-\pi}^{\pi} \left| \int_{\Gamma} G_{im}(\alpha, \varphi) \tilde{X}_m(\alpha)\, d\alpha \right|^2 d\varphi} <$$
$$< h^{-1} \sqrt{2\pi}\theta_{11}(1 + 3V_{s1}^2 V_{p1}^{-2}), \quad \left| \int_0^{\infty} R X_1(R)\, dR \right| < 1. \tag{1.8}$$

Together with condition (1.8), the continuity of the functions which determine the elements of the matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{S}$ and the vectors $\tilde{\mathbf{T}}$, $\hat{\mathbf{V}}$ makes it possible to find functions Y_j, X_j, R_j, q_j ($j = 1, 2$) in the form of expansions in the small parameter h^{-1} [4]:

$$f = f^{(0)} + h^{-1} f^{(1)} + \ldots, \quad f = Y_j,\ X_j,\ R_j,\ q_j \ (j = 1, 2). \tag{1.9}$$

Here, the contour integrals are evaluated by asymptotic methods with the accuracy necessary to find each subsequent term in expansion (1.9). To determine $q_j^{(k)}$, at each step we need to use the method of successive approximations to solve singular integral equation (1.7). The right side of this equation depends on the preceding terms of the expansion $q_j^{(k-1)}$, $X_j^{(k-1)}$, $R_j^{(k-1)}$, $Y_j^{(k-1)}$. We find the functions $q_j^{(0)}$ by solving the contact problem for a vibrating die on a two-layer medium without a cavity. The equation is solved by using the method of approximate factorization of matrix functions [5].

2. We assume that the die is massive and we represent its mass as m. Considering that the die's motion is due to an applied load which changes by the law (1.1), the equation of motion of die can be written as

$$m\ddot{W}_* = (T_* - Q_*) \exp(-i\omega t),$$
$$T_* = 2\pi a^{-2} p R_0, \quad Q_* = 2\pi \mu_1 a \int_0^b R q_1 dR . \tag{2.1}$$

Alternatively, it can be written as follows in terms of the amplitude functions with allowance for $W_* = W \exp(-i\omega t)$

$$W = (Q - T)/(\theta_{22}^2 M), \quad M = m V_{s2}^2/(\mu_2 a^3), \quad Q = Q_*/(\mu_2 a), \quad T = T_*/(\mu_2 a). \tag{2.2}$$

Proceeding on the basis of the simultaneous solution of Eqs. (1.6), (1.7), and (2.2), we can study the effect of the mass of the die, the location of the cavity in the medium, and other parameters of the problem on the amplitudes of the displacements of points of the medium. Here, we make use of theoretical formulas (1.3)-(1.5).

As an example of calculation of the amplitude—frequency characteristics of points of the medium, we present the dependence of the amplitude values of the displacements W on frequency θ_{22} for different values of the mass of the die and different cavity depths. For the values M = 4 and 26, Figs. 1 and 2 show the behavior of $|W|$ in relation to θ_{22}; the solid line corresponds to $h^{-1} = 0$, while the dashed line corresponds to $h^{-1} = 0.3$ and the dot-dash line corresponds to $h^{-1} = 0.5$; $h_1 = 1$, $V_{S2} = 300$ m/sec, $V_{S1} = 600$ m/sec, $V_{pj} = 2V_{Sj}$ ($j = 1, 2$).

It can be seen from the graphs that the bounded resonance frequency depends only slightly on the position and dimensions of the cavity for a fixed die mass. Meanwhile, the amplitude of the bounded resonance is appreciably dependent both on the mass of the die and on the size and depth of the cavity, given the above-noted restrictions. Here, while the dependence of the frequency of bounded resonance on the mass of the die is of the same character as when the cavity is absent [6], the presence of the cavity nearly always leads to a reduction in the amplitude of the first resonance — regardless of the mass of the die.

With an increase in the parameter h^{-1} ($h^{-1} \in (0; 0.5)$) for any M, the reduction in the amplitude of the first resonance is monotonic and can alter the pattern of behavior only when the cavity approaches the plane z = 0 ($h^{-1} \in (0.5; 1)$) and Rayleigh waves begin to have a significant effect. Study of the second and subsequent resonances leads to the conclusion that the presence of a cavity can lead to either a decrease or an increase in the maximum

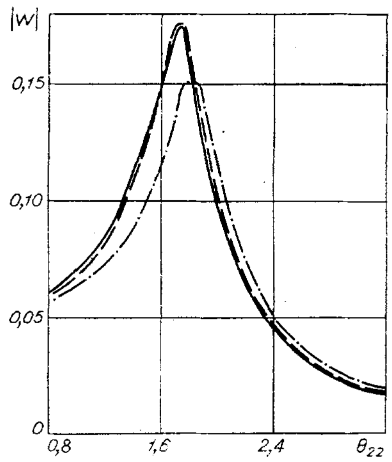

Fig. 1

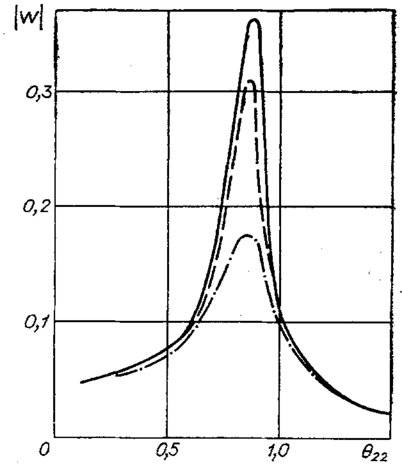

Fig. 2

amplitude. Here, the change in maximum amplitude will be proportional to the change in the amplitude of the first resonance relative to the corrected frequency of the corresponding resonance.

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